Calculate the Maximum Power Capacity of a Heatpipe

entropy

CALCULATE THE MAXIMUM POWER CAPACITY OF A HEATPIPE

Although heat pipes are a very effective conductor of heat, there is a limit to how much thermal power they can carry. A heat pipe relies on a pump cycle of vapor to the condenser end, and condensed liquid back to the evaporator end, overcoming viscous friction and gravitational forces along the way. This pumping pressure is driven by capillary action [1].

The maximum power a heat pipe can dissipate, Q_{max}, is dependent on the available capillary pumping pressure, $\Delta P_{c,\max}$, which must be greater than the total pressure drop in the pipe. The total pressure drop is made up of three components:

(i) The pressure drop ΔP_l required to return the liquid from the condenser to the evaporator against viscous friction.
(ii) The pressure drop ΔP_v necessary to cause the vapor to flow from the evaporator to the condenser against viscous friction.
(iii) The pressure due to the hydrostatic head, ΔP_g which may be zero, positive or negative, depending on the inclination of the heat pipe

$$\Delta P_{c,\max} \geq \Delta P_l + \Delta P_v + \Delta P_g \qquad (1)$$

If this condition is not met, the wick will dry out in the evaporator region and the heat pipe will not operate. The capillary limit will therefore determine the maximum power carrying capability of the pipe over much of its operating range [2]. There are a number of other limitations to the performance of a pipe over its operating life, however this paper will look into more detail at the Capillary Limit only.

CAPILLARY PUMPING PRESSURE

The capillary pumping pressure, ΔP_c , is dependent on the liquid surface tension of the carrier fluid, δ_{lv}, the effective radius of the wick, r_C, and the contact angle the fluid makes with the wick surface, θ.

SURFACE TENSION

Surface tension is an important phenomenon in heat pipes. Surface tension, δ_{lv}, is a property of a liquid-vapor interface and measured in N/m. Consider a vapor bubble at rest in a liquid Figure 1(a), where the pressure inside the bubble is denoted as PI and the outside pressure is denoted as PII. We can construct a free body diagram by cutting the bubble in half where the force F, due to surface tension, δ_{lv}, is acting along the cut interface circumference of length $2\pi r$.

$$F = \delta_{lv}(2\pi r) \qquad (2)$$

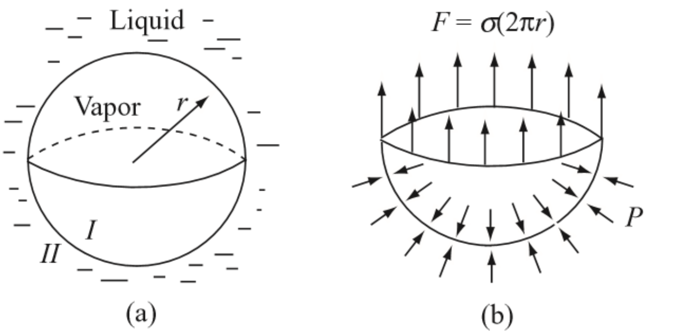

FIGURE 1 - (A) A VAPOR BUBBLE IN LIQUID, (B) CUTAWAY OF A HALF BUBBLE

Given the force due to surface tension, F, is acting in the vertical direction only, we must use an equivalent expression for the vertical force exerted by PI and PII. This can be done by using the cross-sectional area of the cut interface, πr^2, to construct a vertical force component of PI, and PII. Since the bubble is at equilibrium, all the upward vertical forces must balance with the downward vertical forces:

$$F + PII(\pi r^2) = PI(\pi r^2) \qquad (3)$$

Combining Equations (2) and (3) results in the well-known "*Young-Laplace equation*" for a vapor bubble in a fluid [3]:

$$PI - PII = \frac{2\delta_{lv}}{r} \qquad (4)$$

This can be extended to a soap bubble in air (which has two surfaces both inner and outer) and used to calculate δ_{lv} through the "*Maximum Bubble Pressure Method*"- see Equation (22) in Appendix A for details.

CAPILLARY ACTION

The phenomenon of the capillary effect can be explained by considering cohesive forces (the forces between like molecules, such as water and water) and adhesive forces (the forces between unlike molecules, such as water and glass). For example, water molecules in a glass capillary tube (see Figure 2) are more strongly attracted to the glass molecules than they are to other water molecules, and thus water tends to rise along the glass surface. The opposite occurs for mercury, which causes the liquid surface near the glass wall to be suppressed [4].

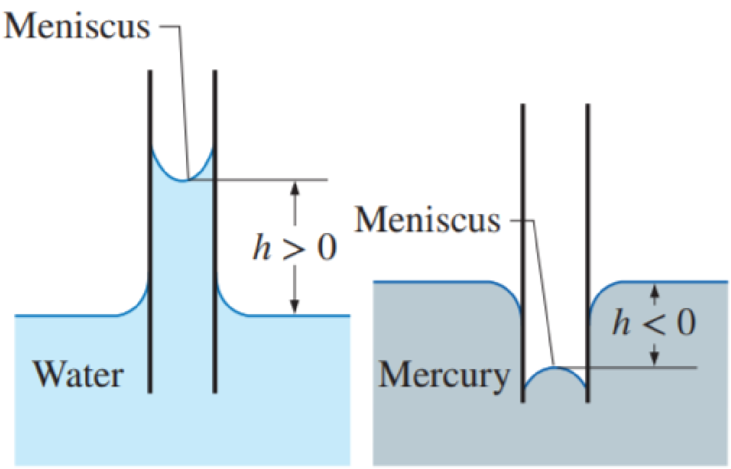

FIGURE 2 - THE CAPILLARY RISE OF WATER AND CAPILLARY DEPRESSION OF MERCURY IN A GLASS TUBE

Due to the additional interactions of adhesion and cohesion, the surface tension at the meniscus will include a horizontal force component, and thus a different vertical component depending on the contact angle, θ, presented in Figure 3 below. When $0 < \theta < 90°$, a liquid is termed wetting (hydrophilic). When $90 < \theta < 180°$∘, the liquid is termed nonwetting (hydrophobic). Wetting interfaces will cause a capillary rise, fundamental to the capillary action of a heat pipe [3].

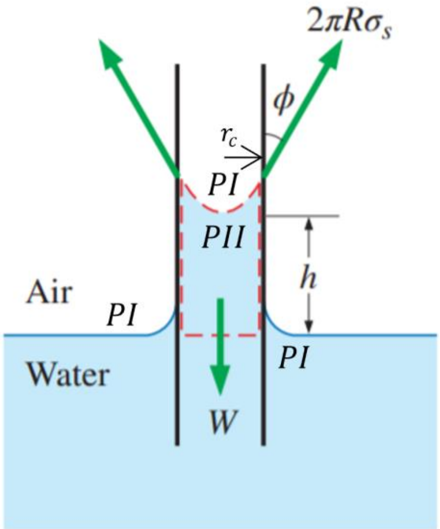

FIGURE 3 - FREE BODY DIAGRAM OF FORCES DUE TO SURFACE TENSION AND FLUID WEIGHT

From Figure 3 above, the vertical component of the force due to surface tension in a capillary tube is:

$$F = \delta_{lv}(2\pi r_c)\cos\theta \qquad (5)$$

The force due to the surface tension, F, is opposed by the weight of the of the liquid above the water surface level. With h representing the height of the fluid above the water surface level (Figure 3), the weight of the fluid W, can be described in terms of mass (m), density (ρ), volume (V) and acceleration to gravity (g) [3].

$$W = mg = \rho V g = \rho(\pi {r_c}^2 h)g \qquad (6)$$

Given the system is at equilibrium, the upward force due to surface tension, F, must equal the downward force due to liquid weight, W.

$$W = F \qquad (7)$$

$$\rho(\pi {r_c}^2 h)g = \delta_{lv}(2\pi r_c)\cos\theta \qquad (8)$$

Rearranging the above,

$$\rho g h = \frac{2\delta_{lv}}{r_c} cos\theta \qquad (9)$$

Solving for h, gives the capillary rise to be [4]:

$$h = \frac{2\delta_{lv}}{\rho g r_c} cos\theta \qquad (10)$$

It can be shown by Equation (24), that the pressure drop, $PI - PII\ (\Delta P_c) = \rho g h$. Combining Equations (9) and (24) we have an expression for the capillary pumping pressure:

$$\Delta P_c = \frac{2\delta_{lv}}{r_c}\cos\theta \qquad (11)$$

r_c = capillary radius

As presented above, the smaller the capillary radius, r_c, the greater the pumping pressure. The capillary radius is normally the pore radius, however there are some exceptions for unusually shaped wicks, such as triangular groove or trapezoidal wicks. Manufacturing data should be consulted for the r_c adjustment factor for these wicks.

TOTAL PRESSURE DROP ALONG THE PIPE

The three pressure drops attributed to losses in the pipe, as identified in Equation (1), are distinguished by design engineers to perfect the operation of a pipe for the correct application.

PRESSURE DROP DUE TO LIQUID PHASE

The liquid pressure drop, ΔP_l and flow rate for a wick structure is described by Darcy's law [2]:

$$\Delta P_l = \frac{\mu_l l_{eff} Q}{\rho_l K_l A_l L} \qquad (12)$$

Where,
ρ_l = liquid density (kg/m^3)
Q= heat transfer rate (W)
l_{eff} = effective heat-pipe length (m)
K_l = constant of permeability of the liquid wick (m^2)
μ_l = dynamic viscosity of the fluid (Pa.s)
A_l= fluid flow cross sectional area of wick (m^2)
L = latent heat of vaporisation (J/kg)

Since mass flow will vary in both the evaporator and the condenser region, an effective length rather than the geometrical length must be used for these regions [2]. This value assumes that the vapor flow reduces linearly towards the end of the pipe across the length of the evaporator or condenser, so it is assumed the functional length is half.

$$l_{eff} = l_a + \frac{l_e + l_c}{2} \qquad (13)$$

Where,
l_a = length of adiabatic region (m)
l_e = length of evaporator region (m)
l_c = length of condenser region (m)

The fluid flow cross section of the wick area is calculated as such:

$$A_l = \pi\left(r_{pipe}^2 - r_{vapor}^2\right)\varepsilon \qquad (14)$$

Where,
ε = volume fraction of the liquid phase in the wick $0 < \varepsilon < 1$

The permeability, K_l, measures how easy it is for fluid to flow through the wick and is directly related to pore radius. For a sintered wick, Andersons curve Equation (21) below, can be used. A method of experimentally determining the permeability using Darcy's Law is presented in **Error! Reference source not found.**.

PRESSURE DROP DUE TO VAPOR PHASE

The pressure drop can be calculated using the following [3]:

$$\Delta P_v = \frac{C(f_v Re_v)\mu_v l_{eff} Q}{2r_v^2 A_v \rho_v L} \qquad (15)$$

Where,
f_v = is the friction coefficient of the vapor (dimensionless)
Re_v= Reynolds number of the vapor (dimensionless)
C= correction factor due to vapor speed (dimensionless)
ρ_v = vapor density (kg/m^3)
μ_v = dynamic viscosity of the fluid (Pa.s)
A_v = cross sectional area of vapor space (m^2)
r_v = hydraulic radius of vapor space (m^2)
L = latent heat of vaporisation (J/kg)

The following experimentally determined values have been presented in the literature [3], when $Re_v < 2300$ (laminar) and vapor speed $< Mach\ 0.2$:

$$(f_v)(Re_v) = 16 \qquad (16)$$
$$C = 1 \qquad (17)$$

The vapor-phase pressure drop is usually a very small fraction (<1%) of the liquid-phase pressure drop and so is typically neglected when estimating the maximum power capacity. A worked example of this is provided in Appendix B.

PRESSURE DROP DUE TO HYDROSTATIC HEAD

The pressure difference, ΔP_g, due to the hydrostatic head of liquid may be positive, negative or zero, depending on the relative positions of the condenser and evaporator. The pressure difference for the hydrostatic head can be determined from the following equation [2]:

$$\Delta P_g = \rho g l_{eff}\ sin\theta \qquad (18)$$

Where,
g = acceleration due to gravity 9.81 (m/s^2)
θ = angle between the heat-pipe and the horizontal (°)

If the evaporator section lies above the condenser section, ΔP_g represents a pressure drop or a positive sign in Equation (18) above. The reverse orientation will yield a pressure rise or a negative sign in Equation (18). This is presented in Figure 4 below, where θ is the inclination angle of the heat pipe with respect to horizontal.

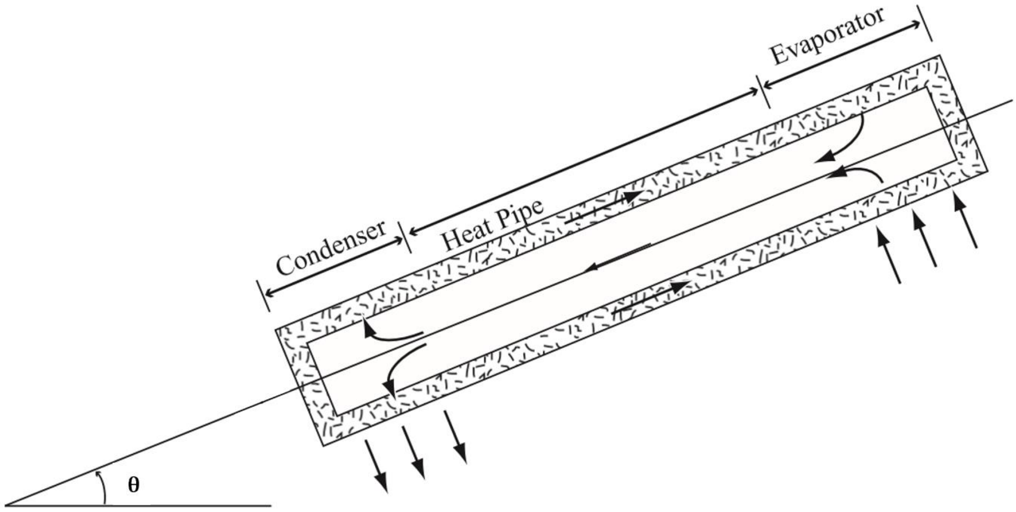

FIGURE 4 - ORIENTATION OF A HEAT PIPE FOR HYDROSTATIC HEAD CALCULATION [3]

DETERMINING MAXIMUM POWER

Using the standard pressure balance from Equation (1) and sufficient information from pipe design as calculated in the previous section, the design engineer can determine the maximum thermal carrying capability of the pipe.

Neglecting the vapor pressure drop due to its insignificant contribution to the overall losses, Equation (1) is equivalent to:

$$\frac{2\delta\cos\theta}{r_c} = \frac{\mu l_{eff} Q_{max}}{\rho_l K_l A_l L} + \rho g l_{eff}\sin\theta \qquad (19)$$

And rearranging provides a function for Q_{max}:

$$Q_{max} = \frac{\rho K_l A_l}{\mu l_{eff}}\left(\frac{2\delta\cos\theta}{r_c} - \rho g l_{eff}\sin\theta\right) L \qquad (20)$$

The value of this function is that it allows the design engineer to observe the direct effect of each dependent factor of pipe design.

OPTIMIZING THE WICK

The ideal heat pipe wick would have a small pore size, r_c, to maximize the capillary pumping pressure (Equation (11)), and a high permeability, K_l, to minimize the liquid pressure drop (Equation (12)). Unfortunately, these two properties are mutually exclusive. As the pore size decreases, the permeability also decreases, much faster and so the wick selected for any given heat pipe is a trade-off between pore size and permeability. The Anderson curve, Figure 5, provides an approximate curve fit Equation (21), derived from experiments measuring the relationship between pore size and permeability for sintered wicks [5]:

$$K_l = 0.125{r_c}^{2.207} \qquad (21)$$

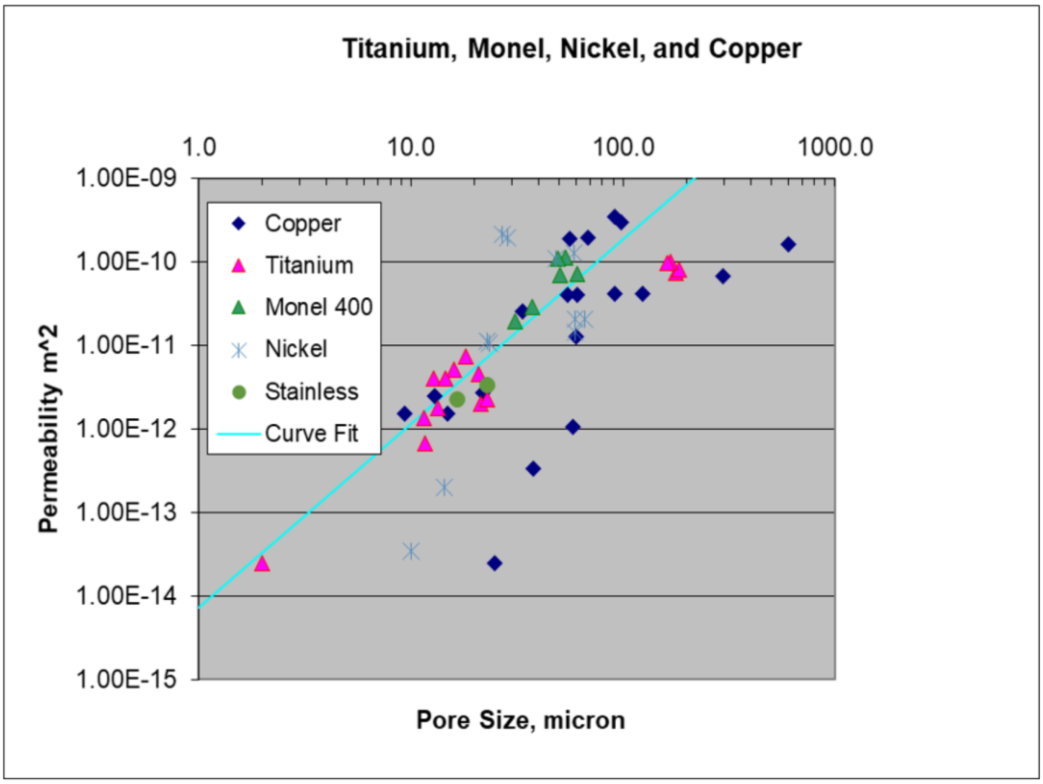

Figure 5 - Anderson curve relating pore size to permeability [5]

OPTIMIZING PIPE GEOMETRY

Option to decrease the liquid pressure drop in Equation (12) would be to increase the fluid flow cross sectional area of the wick, A_l. This can be done by increasing the cross sectional area of wick, $\pi(r_{pipe}^2 - r_{vapor}^2)$, or the volume fraction of the liquid phase in the wick, ε (Equation (14)). Consequently, a thicker and more porous wick will increase the thermal resistance of the heat pipe.

Finally decreasing the effective length, l_{eff}, will decrease the pressure drop Equation (12), however this will reduce the conductive surface area of the evaporator and condenser. As a result, this will also increase the thermal resistance of the heat pipe. Further details on calculating the thermal resistance of the heat pipe can be found in Entropy's technical paper on "*How to Calculate and Simulate the Performance of a Heat Pipe*".

These variables are levers at the disposal of the designer to optimise the heat pipe performance for the application. Although, attention must always be paid to the knock-on effects of any adjustment.

FURTHER LIMITATIONS TO TWO PHASE COOLING

This technical paper has detailed how the available capillary pumping pressure dictates the limit of thermal power capacity in a heat pipe. Although the capillary pumping pressure is generally considered the limiting factor, there are other limitations to be considered such as viscous, sonic, entrainment, and boiling limits which are outside the scope of this document.

APPENDIX A

SOAP BUBBLE *YOUNG-LAPLACE* EQUATION

The extra factor of 2 in the force balance for the soap bubble is due to the existence of a soap film with two surfaces (inner and outer surfaces) and thus two circumferences in the cross section.

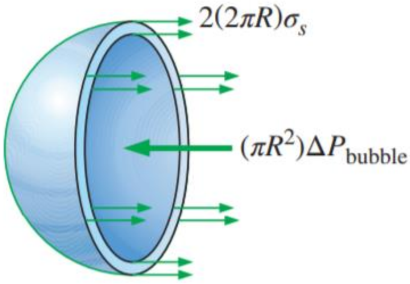

FIGURE 6 - SURFACE TENSION ON SOAP BUBBLE

Using the same method as described for the vapor bubble, the "*Young-Laplace equation*" for a soap bubble in air is presented as:

$$PI - PII = \frac{4\delta_{lv}}{r} \qquad (22)$$

MAXIMUM BUBBLE PRESSURE METHOD

The "*Young-Laplace equation*" is typically used in the "*Maximum Bubble Pressure Method*" to determine δ_{lv}. A bubble is generated at the end of a capillary tube as presented in Figure 7 below. Equation (4) states that the maximum pressure difference occurs at the minimum bubble radius. The minimum bubble radius occurs just as it passes through the end of the tube, equalling the capillary radius. Given the pressure difference, $P_{max} - P_{min}$, and capillary tube radius r_{cap} are known, δ_{lv} can be calculated.

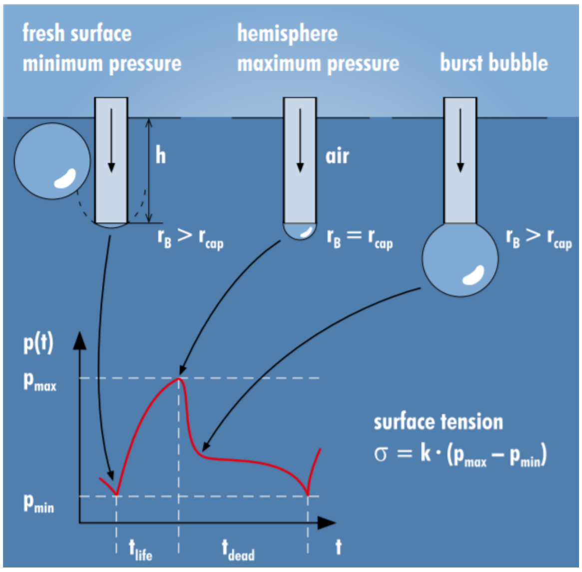

FIGURE 7 - MAXIMUM BUBBLE PRESSURE METHOD

BERNOULLI EQUATION FOR PRESSURE CHANGE IN CAPILLARY TUBE

Referring to the Bernoulli Equation (23) below, the fluid velocity of the system in Figure 3 is constant $v_1 = v_2$, so there is no dynamic pressure change, and thus these terms cancel. Setting the water surface level at $h_1 = 0$, it can be shown that the static pressure drop between the two points is:

$$PI + \cancel{\rho g h_1} + \cancel{\frac{1}{2} v_1^2} = PII + \rho g h_2 + \cancel{\frac{1}{2} v_2^2} \qquad (23)$$

Hence,

$$PI - PII = \rho g h \qquad or \qquad \Delta P_c = \rho g h \qquad (24)$$

TESTING RIG TO CALCULATE PERMEABILITY OF A WICK

Given [2]:

$$Q = \dot{m} L \quad (25) \qquad and \qquad \dot{m} = \rho \dot{v} \quad (26)$$

Where,
$\dot{m}$= mass flow rate (kg/s)
$\dot{v}$= volume flow rate (m^3/s)

Combining Equations (12), (25) and (26), assuming all other variables are measured, the following can be used to solve for wick permeability:

$$K_l = \frac{\mu_l l_{eff} \dot{v}}{\Delta P_l A_l} \qquad (27)$$

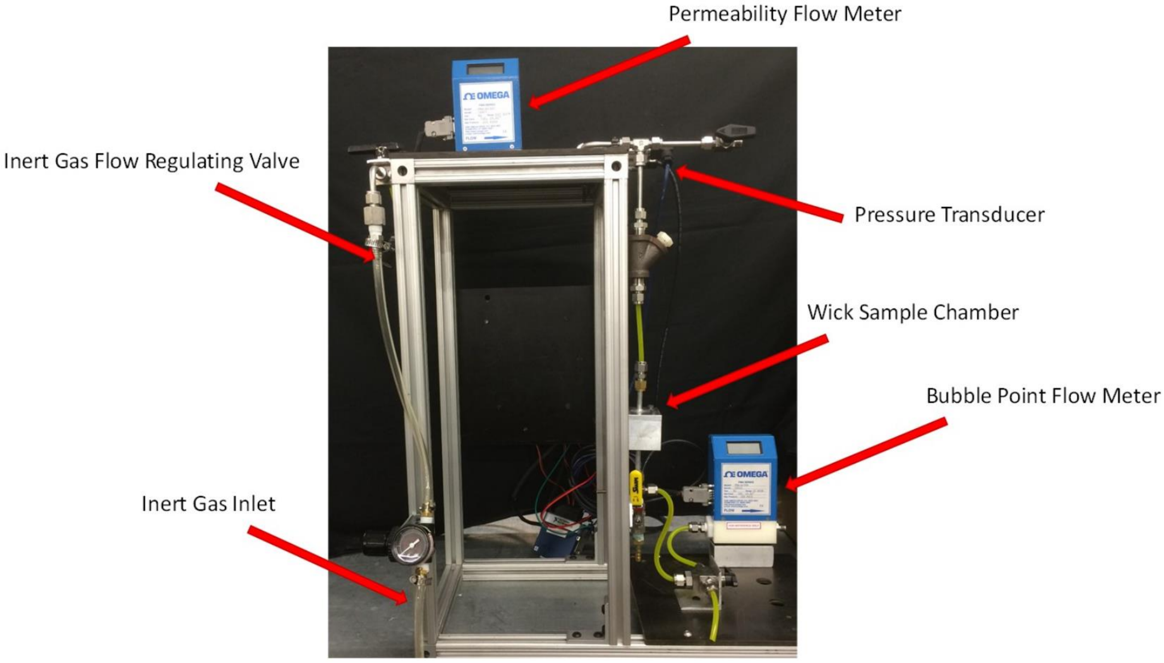

FIGURE 8 - TEST EQUIPMENT SET UP TO MEASURE PERMEABILITY [5]

APPENDIX B

WORKED EXAMPLE OF MAXIMUM POWER

A sintered wick heat pipe for an electronics application uses water as the working fluid. The working fluid is perfectly wetting with a contact angle $\theta = 0°$. The evaporator is $2x10^{-2}m$ long, the condenser is $3x10^{-2}m$ long, and the heat pipe has no adiabatic section, as shown in Figure 9. The diameter of the heat pipe $d_p = 4x10^{-3}m$, vapor space diameter $d_v = 3x10^{-3}m$ and pore radius $r_c = 5x10^{-5}m$. The volume fraction of the wick, $\varepsilon = 0.3$. It can be assumed that the flow in the vapor section is laminar where $Re < 2300$, and the speed of the vapor is $< Mach\ 0.2$. If the heat pipe operates at 80°C in a horizontal position, determine the maximum power due to capillary limitation. The properties of working fluid are found below in Table 1.

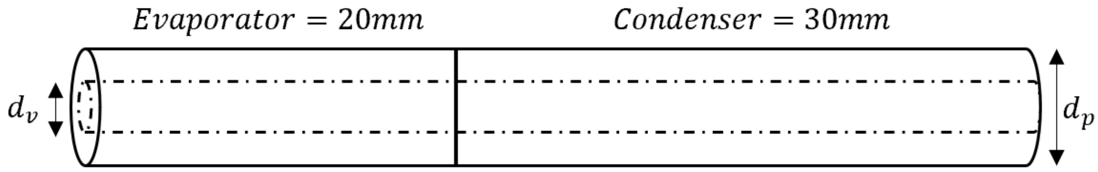

FIGURE 9 -HEAT PIPE

TABLE 1 - PROPERTIES OF LIQUID WATER AND VAPOR

Water									
Temp °C	Latent heat kJ/kg	Liquid density kg/m³	Vapour density kg/m³	Liquid thermal conductivity W/m °C	Liquid viscos. cP	Vapour viscos. cP × 10²	Vapour press. bar	Vapour specific heat kJ/kg °C	Liquid surface tension N/m × 10²
20	2448	998.2	0.02	0.603	1.00	0.96	0.02	1.81	7.28
40	2402	992.3	0.05	0.630	0.65	1.04	0.07	1.89	6.96
60	2359	983.0	0.13	0.649	0.47	1.12	0.20	1.91	6.62
80	2309	972.0	0.29	0.668	0.36	1.19	0.47	1.95	6.26
100	2258	958.0	0.60	0.680	0.28	1.27	1.01	2.01	5.89
120	2200	945.0	1.12	0.682	0.23	1.34	2.02	2.09	5.50
140	2139	928.0	1.99	0.683	0.20	1.41	3.90	2.21	5.06
160	2074	909.0	3.27	0.679	0.17	1.49	6.44	2.38	4.66
180	2003	888.0	5.16	0.669	0.15	1.57	10.04	2.62	4.29
200	1967	865.0	7.87	0.659	0.14	1.65	16.19	2.91	3.89

Referring to the governing equation below, the hydrostatic force due to inclination can be ignored because the heat pipe is operating in a horizontal position.

$$\Delta P_{c,\max} \geq \Delta P_l + \Delta P_v + \cancel{\Delta P_g} \qquad (1)$$

The capillary pumping pressure, remember, is defined by:

$$\Delta P_c = \frac{2\delta_{lv}}{r_c}\cos\theta \qquad (11)$$

Using the given values and referencing Table 1:

$$\Delta P_c = \frac{2\delta_{lv}}{r_c}\cos\theta = \frac{2(6.26x10^{-2})}{5x10^{-5}}\cos(0) = 2{,}504\ Pa$$

The liquid pressure drop is defined by:

$$\Delta P_l = \frac{\mu_l l_{eff} Q}{\rho_l K_l A_l L} \qquad (12)$$

The fluid flow cross sectional area of the wick is calculated as:

$$A_l = \pi\left(r_{pipe}^2 - r_{vapor}^2\right)\varepsilon = 3.14((2x10^{-3})^2 - (1.5x10^{-3})^2)0.3 = 1.64x10^{-6}m^2$$

The permeability of the wick is calculated using Anderson’s Equation (21):

$$K_l = 0.125{r_c}^{2.207} = 0.125(5x10^{-5})^{2.207} = 4x10^{-11}m^2$$

So,

$$\Delta P_l = \frac{(3.6x10^{-4})(2.5x10^{-2})Q}{(972)(4x10^{-11})(1.64x10^{-6})(2.3x10^{6})} = 61.36(Q)\ Pa$$

The vapor pressure drop is defined by:

$$\Delta P_v = \frac{C(f_v Re_v)\mu_v l_{eff} Q}{2r_v^2 A_v \rho_v L} \qquad (15)$$

Given when $Re_v < 2300$ (laminar) and vapor speed $< Mach\ 0.2$,

$$(f_v Re_v) = 16 \qquad (28)$$
$$C = 1 \qquad (29)$$

Knowing the hydraulic radius for a circular tube is simply the vapor radius, the vapor drop is calculated as:

$$\Delta P_v = \frac{(1)(16)(1.19x10^{-5})(2.5x10^{-2})Q}{2(1.5x10^{-3})^2(3.14(1.5x10^{-3})^2)(2.9x10^{-1})(2.3x10^6)} = 0.2\ (Q)\ Pa$$

Hence,

$$2{,}504 = 61.36(Q) + 0.2(Q)$$

$$Q_{max} = 40.67\ W$$

The vapor pressure drop in this example of a typical heat pipe is <1% of the overall pressure drop, and thus is normally neglected to reduce complexity in estimating Q_{max}.

Using instead the Q_{max} approximation from Equation (20) which neglects the vapor pressure drop:

$$Q_{max} = \frac{\rho K_l A_l}{\mu_l l_{eff}}\left(\frac{2\delta\cos\theta}{r_c} - \rho g l_{eff}\sin\theta\right)L \qquad (20)$$

$$Q_{max} = \frac{(972)(4x10^{-11})(1.64x10^{-6})}{(3.6x10^{-4})(2.5x10^{-2})}\left(\frac{2(6.26x10^{-2})}{(5x10^{-5})} - 0\right)(2.3x10^6) = 40.8\ W$$

Which gives a near negligible difference for most heat pipe applications.

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